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# Statistical Techniques in Business \& Economics 

Eighteenth
Edition

# Statistical Techniques in BUSINESS ECONOMICS 

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# Statistical Techniques in BUSINESS ECONOMICS 

EIGHTEENTH EDITION

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## STATISTICAL TECHNIQUES IN BUSINESS \& ECONOMICS

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## DEDICATION

To Jane, my wife and best friend, and our sons, their wives, and our grandchildren: Mike and Sue (Steve and Courtney), Steve and Kathryn (Kennedy, Jake, and Brady), and Mark and Sarah (Jared, Drew, and Nate).

Douglas A. Lind

To Margaret Marchal Nicholson and Andrea.
William G. Marchal

To my wonderful family: Barb, Hannah, and Isaac.
Samuel A. Wathen

Over the years, we received many compliments on this text and understand that it's a favorite among students. We accept that as the highest compliment and continue to work very hard to maintain that status.

The objective of Statistical Techniques in Business and Economics is to provide students majoring in management, marketing, finance, accounting, economics, and other fields of business administration with an introductory survey of descriptive and inferential statistics. To illustrate the application of statistics, we use many examples and exercises that focus on business applications, but also relate to the current world of the college student. A previous course in statistics is not necessary, and the mathematical requirement is first-year algebra.

In this text, we show beginning students every step needed to be successful in a basic statistics course. This step-by-step approach enhances performance, accelerates preparedness, and significantly improves motivation. Understanding the concepts, seeing and doing plenty of examples and exercises, and comprehending the application of statistical methods in business and economics are the focus of this book.

The first edition of this text was published in 1967. At that time, locating relevant business data was difficult. That has changed! Today, locating data is not a problem. The number of items you purchase at the grocery store is automatically recorded at the checkout counter. Phone companies track the time of our calls, the length of calls, and the identity of the person called. Credit card companies maintain information on the number, time and date, and amount of our purchases. Medical devices automatically monitor our heart rate, blood pressure, and temperature from remote locations. A large amount of business information is recorded and reported almost instantly. CNN, USA Today, and MSNBC, for example, all have websites that track stock prices in real time.

Today, the practice of data analytics is widely applied to "big data." The practice of data analytics requires skills and knowledge in several areas. Computer skills are needed to process large volumes of information. Analytical skills are needed to evaluate, summarize, organize, and analyze the information. Critical thinking skills are needed to interpret and communicate the results of processing the information.

Our text supports the development of basic data analytical skills. In this edition, we added a new section at the end of each chapter called Data Analytics. As you work through the text, this section provides the instructor and student with opportunities to apply statistical knowledge and statistical software to explore several business environments. Interpretation of the analytical results is an integral part of these exercises.

A variety of statistical software is available to complement our text. Microsoft Excel includes an add-in with many statistical analyses. Megastat is an add-in available for Microsoft Excel. Minitab and JMP are stand-alone statistical software packages available to download for either PC or MAC computers. In our text, Microsoft Excel, Minitab, and Megastat are used to illustrate statistical software analyses. The text now includes references or links to Excel tutorials in Connect. These provide users with clear demonstrations using statistical software to create graphical and descriptive statistics and statistical analyses to test hypotheses. We use screen captures within the chapters, so the student becomes familiar with the nature of the software output.

Because of the availability of computers and software, it is no longer necessary to dwell on calculations. We have replaced many of the calculation examples with interpretative ones, to assist the student in understanding and interpreting the statistical results. In addition, we place more emphasis on the conceptual nature of the statistical topics. While making these changes, we still continue to present, as best we can, the key concepts, along with supporting interesting and relevant examples.

## WHAT'S NEW IN THE EIGHTEENTH EDITION?

We made several significant improvements to our text. First, based on reviewer suggestions, Chapter 18, now titled "Forecasting with Time Series Analysis," is completely rewritten. The chapter shows how to create forecasting models that mimic trend and seasonal time series patterns. The chapter shows how to apply simple moving averages, simple exponential smoothing, regression, and seasonal indexing to create the models. Forecasting error, using the mean absolute deviation, is included for every forecasting model. In addition, the chapter includes a discussion of autocorrelation and the Durban-Watson statistic. While there are a few similarities to past editions, all examples/ solutions and self-reviews are new; all have associated data files. All 36 exercises are new; 28 have associated data files and require statistical software to complete the responses. Some of the data files were created by the authors; others use real data sourced from the U.S. Census Bureau website, www.census.gov. All data files are available in Connect. We hope that this chapter satisfies the need of users to include time series forecasting in their courses.

Second, the text now uses an excellent set of Excel tutorials to demonstrate how to use Excel to perform the statistical analyses in the text. Rather than referring to a set of written procedures, users will be able to view well-organized presentations that clearly demonstrate how to use the various statistical tools, functions, and analyses in Excel. The references to the tutorials are indicated by a unique icon placed in the left margin and aligned with the procedure or analysis in the text. In the eBook, these icons will be directly linked to the tutorials. Textbook users will access the tutorials through Connect.

Third, Chapter 8 now starts with a brief discussion of the research process to establish a context for sampling and data collection. It also includes comments on ethics and biased sampling. After the description of sampling methods, a new section, "Sample Mean as a Random Variable," demonstrates the effect of random sampling on the sample mean followed by the section, "Sampling Distribution of the Sample Mean." The standard error of the sampling distribution is now featured in a new section. The "sampling error" concept continues to be a key item in the chapter's discussion.

Fourth, starting in Chapter 9, many exercises have been restructured with multiitem responses. Reformulating these exercises will provide users with more direction to understanding the details of a particular statistical technique. As an example, compare Chapter 11, exercise 20 before, on the right, and after, on the left.
20. FILE The federal government recently granted funds for a special program designed to reduce crime in high-crime areas. A study of the results of the program in eight high-crime areas of Miami, Florida, yielded the following results.


Has there been a decrease in the number of crimes since the inauguration of the program? Use the .01 significance level. Estimate the $p$-value.
20. FILE The federal government recently granted funds for a special program designed to reduce crime in high-crime areas. A study of the results of the program in eight high-crime areas of Miami, Florida, yielded the following results.


Has there been a decrease in the number of crimes since the inauguration of the program? Use the .01 significance level.
a. State the null and alternate hypotheses.
b. What is the $p$-value?
c. Is the null hypothesis rejected?
d. What is the conclusion indicated by the analysis?

## HOW ARE CHAPTERS ORGANIZED TO ENGAGE STUDENTS AND PROMOTE LEARNING?

## Chapter Learning Objectives

Each chapter begins with a set of learning objectives designed to provide focus for the chapter and motivate student learning. These objectives, located in the margins next to the topic, indicate what the student should be able to do after completing each section in the chapter.

- MERRILL LYNCH recently completed a study of online investment portfolios for a sample of clients. For the 70 participants in the study, organize these data into a frequency distribution. (See Exercise 43 and LO2-3.)


## LEARNING OBJECTIVES

When you have completed this chapter, you will be able to:
LO2-1 Summarize qualitative variables with frequency and relative frequency tables.
LO2-2 Display a frequency table using a bar or pie chart.
LO2-3 Summarize quantitative variables with frequency and relative frequency distributions.
LO2-4 Display a frequency distribution using a histogram or frequency polygon.

## Chapter Opening Exercise

A representative exercise opens the chapter and shows how the chapter content can be applied to a real-world situation.

## Introduction to the Topic

Each chapter starts with a review of the important concepts of the previous chapter and provides a link to the material in the current chapter. This step-by-step approach increases comprehension by providing continuity across the concepts.

## Example/Solution

After important concepts are introduced, a solved example is given. This example provides a how-to illustration and shows a relevant business application that helps students answer the question, "How can I apply this concept?"


## EXAMPLE

The service departments at Tionesta Ford Lincoln and Sheffield Motors Inc., two of the four Applewood Auto Group dealerships, were both open 24 days last month. Listed below is the number of vehicles serviced last month at the two dealerships. Construct dot plots and report summary statistics to compare the two dealerships.

| Tionesta Ford Lincoln |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| 23 | 33 | 27 | 28 | 39 | 26 |
| 30 | 32 | 28 | 33 | 35 | 32 |
| 29 | 25 | 36 | 31 | 32 | 27 |
| 35 | 32 | 35 | 37 | 36 | 30 |

## Self-Reviews

Self-Reviews are interspersed throughout each chapter and follow Example/Solution sections. They help students monitor their progress and provide immediate reinforcement for that particular technique. Answers are in Appendix D.


| 7.69 | 7.72 | 7.80 | 7.86 | 7.90 | 7.94 | 7.97 | 8.06 | 8.09 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(a) What is the median weight?
(b) Determine the weights corresponding to the first and third quartiles.

## Statistics in Action

Statistics in Action articles are scattered throughout the text, usually about two per chapter. They provide unique, interesting applications and historical insights in the field of statistics.

## Al <br> STATISTICS IN ACTION <br> If you wish to get some attention at the next gathering you attend, announce that you believe that at least two people present were born on the same date-that is, the same day of the year but not necessarily the same year. If there are 30 people in the room, the probability of a duplicate is .706 . If there

## Definitions

Definitions of new terms or terms unique to the study of statistics are set apart from the text and highlighted for easy reference and review. They also appear in the Glossary at the end of the book.

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.

## Formulas

Formulas that are used for the first time are boxed and numbered for reference. In addition, a formula card is bound into the back of the text that lists all the key formulas.

## Exercises

Exercises are included after sections within the chapter and at the end of the chapter. Section exercises cover the material studied in the section. Many exercises have data files available to import into statistical software. They are indicated with the FILE icon. Answers to the odd-numbered exercises are in Appendix C.

## Computer Output

The text includes many software examples, using Excel, MegaStat ${ }^{\ominus}$, and Minitab. The software results are illustrated in the chapters. Instructions for the software examples are referenced in online tutorials in Connect.

## EXERCISES

> For Exercises 47-52, do the following:
a. Compute the sample variance.
b. Determine the sample standard deviation.
47. Consider these values a sample: $7,2,6,2$, and 3 .
48. The following five values are a sample: $11,6,10,6$, and 7 .
49. FILE Dave's Automatic Door, referred to in Exercise 37, installs automatic garage door openers. Based on a sample, following are the times, in minutes, required to install 10 door openers: $28,32,24,46,44,40,54,38,32$, and 42 .
50. FILE The sample of eight companies in the aerospace industry, referred to in Exercise 38, was surveyed as to their return on investment last year. The results are $10.6,12.6,14.8,18.2,12.0,14.8,12.2$, and 15.6 .

| 좝 APPLEWOOD AUTO GROUP |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | A | B | C | D | E | F | G | H |
| 1 | Age | Profit | Location | Vehicle-Type | Previous |  | Profi |  |
| 2 | 21 | \$1,387 | Tionesta | Sedan | 0 |  |  |  |
| 3 | 23 | \$1,754 | Sheffield | SUV | 1 |  | Mean | 1843.17 |
| 4 | 24 | \$1,817 | Sheffield | Hybrid | 1 |  | Standard Error | 47.97 |
| 5 | 25 | \$1,040 | Sheffield | Compact | 0 |  | Median | 1882.50 |
| 6 | 26 | \$1,273 | Kane | Sedan | 1 |  | Mode | 1915.00 |
| 7 | 27 | \$1,529 | Sheffield | Sedan | 1 |  | Standard Deviation | 643.63 |
| 8 | 27 | \$3,082 | Kane | Truck | 0 |  | Sample Variance | 414256.61 |
| 9 | 28 | \$1,951 | Kane | SUV | 1 |  | Kurtosis | -0.22 |
| 10 | 28 | \$2,692 | Tionesta | Compact | 0 |  | Skewness | -0.24 |
| 11 | 29 | \$1,342 | Kane | Sedan | 2 |  | Range | 2998 |
| 12 | 29 | \$1,206 | Sheffield | Sedan | 0 |  | Minimum | 294 |
| 13 | 30 | \$443 | Kane | Sedan | 3 |  | Maximum | 3292 |
| 14 | 30 | \$1,621 | Sheffield | Truck | 1 |  | Sum | 331770 |
| 15 | 30 | \$754 | Olean | Sedan | 2 |  | Count | 180 |

## HOW DOES THIS TEXT REINFORCE STUDENT LEARNING?

## BY CHAPTER

## Chapter Summary

Each chapter contains a brief summary of the chapter material, including vocabulary, definitions, and critical formulas.

## Pronunciation Key

This section lists the mathematical symbol, its meaning, and how to pronounce it. We believe this will help the student retain the meaning of the symbol and generally enhance course communications.

## Chapter Exercises

Generally, the end-of-chapter exercises are the most challenging and integrate the chapter concepts. The answers and worked-out solutions for all oddnumbered exercises are in Appendix C. Many exercises are noted with a data file icon in the margin. For these exercises, there are data files in Excel format located on the text's website, www.mhhe.com/Lind18e. These files help students use statistical software to solve the exercises.

## Data Analytics

The goal of the Data Analytics sections is to develop analytical skills. The exercises present a real-world context with supporting data. The data sets are printed in Appendix A and available to download from the text's website www.mhhe.com/Lind18e. Statistical software is required to analyze the data and respond to the exercises. Each data set is used to explore questions and discover findings that relate to a real world context. For each business context, a story is uncovered as students progress from Chapters 1 to 17.

| CHAPTER SUMMARY |  |  |  |
| :---: | :---: | :---: | :---: |
|  | I. A random variable is a numerical value determined by the outcome of an experiment. <br> II. A probability distribution is a listing of all possible outcomes of an experiment and the probability associated with each outcome. <br> A. A discrete probability distribution can assume only certain values. The main features are: <br> 1. The sum of the probabilities is 1.00 . <br> 2. The probability of a particular outcome is between 0.00 and 1.00 . <br> 3. The outcomes are mutually exclusive. <br> B. A continuous distribution can assume an infinite number of values within a specific range. <br> III. The mean and variance of a discrete probability distribution are computed as follows. <br> A. The mean is equal to: |  |  |
|  | B. The variance is equal to: | $\mu=\Sigma[x P(x)]$ | (6-1) |
|  | $\sigma^{2}=\Sigma\left[(x-\mu)^{2} P(x)\right]$ |  | (6-2) |


| PRONUNCIATION KEY |  |  |
| :---: | :---: | :---: |
| SYMBOL | MEANING | PRONUNCIATION |
| $P(A)$ | Probability of $A$ | $P$ of $A$ |
| $P(\sim A)$ | Probability of not $A$ | $P$ of not $A$ |
| $P(A$ and $B)$ | Probability of $A$ and $B$ | $P$ of $A$ and $B$ |
| $P(A$ or $B)$ | Probability of $A$ or $B$ | $P$ of $A$ or $B$ |
| $P(A \mid B)$ | Probability of $A$ given $B$ has happened | $P$ of $A$ given $B$ |
| ${ }_{n} P_{r}$ | Permutation of $n$ items selected $r$ at a time | Pnr |
| ${ }_{n} C_{r}$ | Combination of $n$ items selected $r$ at a time | Cnr |

## CHAPTER EXERCISES

25. According to the local union president, the mean gross income of plumbers in the Sal Lake City area follows the normal probability distribution with a mean of $\$ 45,000$ and a population standard deviation of $\$ 3,000$. A recent investigative reporter for KYAK TV found, for a sample of 120 plumbers, the mean gross income was $\$ 45,500$. At the .10 significance level, is it reasonable to conclude that the mean income is not equal to $\$ 45,000$ ? Determine the $p$-value.
26. FILE Rutter Nursery Company packages its pine bark mulch in 50 -pound bags. From a long history, management knows that the distribution of bag weights is normally distributed with a population standard deviation of 3 pounds per bag. At the end of each day, Jeff Rutter, the production manager, weighs 10 bags and computes the mean weight of the sample. Following are the weights of 10 bags from today's production.

| 45.6 | 47.7 | 47.6 | 46.3 | 46.2 | 47.4 | 49.2 | 55.8 | 47.5 | 48.5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

a. Can Mr. Rutter conclude that the mean weight of the bags is less than 50 pounds? Use the .01 significance level.
b. In a brief report, tell why Mr. Rutter can use the $z$-distribution as the test statistic. c. Compute the $p$-value.
27. A new weight-watching company, Weight Reducers International, advertises that those who join will lose an average of 10 pounds after the first 2 weeks. The population standard deviation is 2.8 pounds. A random sample of 50 people who joined the weight reduction program revealed a mean loss of 9 pounds. At the .05 level of significance,

## DATA ANALYTICS

(The data for these exercises are available at the text website: www.mhhe.com/lind18e.)
60. Refer to the North Valley Real Estate data, which report information on homes sold during the last year.
a. The mean selling price (in $\$$ thousands) of the homes was computed earlier to be $\$ 357.0$, with a standard deviation of $\$ 160.7$. Use the normal distribution to estimate the percentage of homes selling for more than $\$ 500.000$. Compare this to the actual results. Is price normally distributed? Try another test. If price is normally distributed, how many homes should have a price greater than the mean? Compare this to the actual number of homes. Construct a frequency distribution of price. What do you observe?
b. The mean days on the market is 30 with a standard deviation of 10 days. Use

## Software Tutorials

References to tutorials demonstrating how to use Excel to compute various statistics and perform statistical analyses are included throughout the text. See an example of the icon to the right.


Tutorial \#3
in Connect

## Answers to Self-Review

The worked-out solutions to the Self-Reviews are provided in Appendix D.

| 16-6 | Ranks |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Englewood | West Side | Great Northern | Sylvania |
| 17 | 5 | 19 | 7 |  |
| 20 | 1 | 9.5 | 11 |  |
| 16 | 3 | 21 | 15 |  |
| 13 | 5 | 22 | 9.5 |  |
| 5 | 2 | 14 | 8 |  |
| 18 |  |  | 12 |  |

## BY SECTION

## Section Reviews

After selected groups of chapters (1-4, 5-7, 8 and 9, 10-12, 13 and 14,15 and 16 , and 17 and 18), a Section Review is included. Much like a review before an exam, these include a brief overview of the chapters and problems for review.

## Cases

The review also includes continuing cases and several small cases that let students make decisions using tools and techniques from a variety of chapters.

## A REVIEW OF CHAPTERS 1-4

This section is a review of the major concepts and terms introduced in Chapters 1-4. Chapter 1 began by describing the meaning and purpose of statistics. Next we described the different types of variables and the four levels of measurement. Chapter 2 was concerned with describing a set of observations by organizing it into a frequency distribution and then portraying the frequency distribution as a histogram or a frequency polygon. Chapter 3 began by describing measures of location, such as the mean, weighted mean, median, geometric mean, and mode. This chapter also included measures of dispersion, or spread. Discussed in this section were the range, variance, and standard deviation. Chapter 4 included several graphing techniques such as dot plots, box plots, and scatter diagrams. We also discussed the coefficient of skewness, which reports the lack of symmetry in a set of data, and the correlation coefficient.

## CASES

A. Century National Bank

The following case will appear in subsequent review sections. Assume that you work in the Planning Department f the Century National Bank and report to Ms. Lamberg f the Century National Bank and report to Ms. Lamberg. You will need to do some data analysis and prepare a short written report. Remember, Mr. Selig is the president of the bank, so you will want to ensure that your report is complete and accurate. A copy of the data appears in Appendix A. 6 .

Century National Bank has offices in several cities in the Midwest and the southeastern part of the United States. Mr. Dan Selig, president and CEO, would like to know the characteristics of his checking account customers. What is the balance of a typical customer?

How many other bank services do the checking account customers use? Do the customers use the ATM service and, if so, how often? What about debit cards? Who uses them, and how often are they used?
(2) the number of other bank services (a savings account, a certificate of deposit, etc.) the custome uses; (3) whether the customer has a debit card (this is a bank service in which charges are made directly to the customer's account); and (4) whether or not interest is paid on the checking account. The sample includes customers from the branches in Cincinnati, Ohio; Atlanta, Georgia; Louisville, Kentucky; and Erie, Pennsylvania.

1. Develop a graph or table that portrays the checking balances. What is the balance of a typical customer? Do many customers have more than $\$ 2,000$ in their accounts? Does it appear that there is a difference in the distribution of the accounts among the four branches? Around what value do the account balances tend to cluster?
2. Determine the mean and median of the check ing account balances. Compare the mean and the

## Practice Test

The Practice Test is intended to give students an idea of content that might appear on a test and how the test might be structured. The Practice Test includes both objective questions and problems covering the material studied in the section.

## PRACTICE TEST

There is a practice test at the end of each review section. The tests are in two parts. The first part contains several objective questions, usually in a fill-in-the-blank format. The second part is problems. In most cases, it should take 30 to 45 minutes to complete the test. The problems require a calculator. Check the answers in the Answer Section in the back of the book.

Part 1-Objective

1. The science of collecting, organizing, presenting, analyzing, and interpreting data to assist in making effective decisions is called
2. Methods of organizing, summarizing, and presenting data in an informative way are called
3. The entire set of individuals or objects of interest or the measurements obtained from all individuals or objects of interest are called the
4. 
5. 
6. 

$\qquad$

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- Solutions Manual The Solutions Manual, carefully revised by the authors, contains solutions to all basic, intermediate, and challenge problems found at the end of each chapter.
- Test Bank The Test Bank, revised by Wendy Bailey of Troy University, contains hundreds of true/false, multiple choice and short-answer/discussions, updated based on the revisions of the authors. The level of difficulty varies, as indicated by the easy, medium, and difficult labels.
- Powerpoint Presentations Prepared by Stephanie Campbell of Mineral Area College, the presentations contain exhibits, tables, key points, and summaries in a visually stimulating collection of slides.
- Excel Templates There are templates for various end of chapter problems that have been set as Excel spreadsheets-all denoted by an icon. Students can easily download, save the files and use the data to solve end of chapter problems.


## MEGASTAT ${ }^{\circledR}$ FOR MICROSOFT EXCEL ${ }^{\circledR}$

MegaStat ${ }^{\circ}$ by J. B. Orris of Butler University is a full-featured Excel statistical analysis add-in that is available on the MegaStat website at www.mhhe.com/megastat (for purchase). MegaStat works with recent versions of Microsoft Excel ${ }^{\circ}$ (Windows and Mac OS X). See the website for details on supported versions.
Once installed, MegaStat will always be available on the Excel add-ins ribbon with no expiration date or data limitations. MegaStat performs statistical analyses within an Excel workbook. When a MegaStat menu item is selected, a dialog box pops up for data selection and options. Since MegaStat is an easy-to-use extension of Excel, students can focus on learning statistics without being distracted by the software. Ease-of-use features include Auto Expand for quick data selection and Auto Label detect.
MegaStat does most calculations found in introductory statistics textbooks, such as computing descriptive statistics, creating frequency distributions, and computing probabilities as well as hypothesis testing, ANOVA, chi-square analysis, and regression analysis (simple and multiple). MegaStat output is carefully formatted and appended to an output worksheet.

Video tutorials are included that provide a walkthrough using MegaStat for typical business statistics topics. A context-sensitive help system is built into MegaStat and a User's Guide is included in PDF format.

## MINITAB ${ }^{\oplus} /$ SPSS $^{\circledR} / \mathrm{JMP}^{\oplus}$

Minitab ${ }^{\oplus}$, Minitab ${ }^{\circ}$ Express, SPSS ${ }^{\circ}$, and JMP ${ }^{\circ}$ Student Edition are software products that are available to help students solve the exercises with data files. Each software product can be packaged with any McGraw-Hill business statistics text.


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## ENHANCEMENTS TO STATISTICAL TECHNIQUES IN BUSINESS \& ECONOMICS, 18E

## CHAPTER 1 What Is Statistics?

- Updated graphic on data analytics.
- Updated examples.
- Newly revised Ethics and Statistics section.
- New exercise 17 requiring statistical software.
- Updated exercise 19.


## CHAPTER 2 Describing Data: Frequency Tables,

 Frequency Distributions, and Graphic Presentation- Updated examples.
- Updated exercises 33, 38, 47, 48.
- New exercise 44 using extensive data from the pizza industry requiring statistical software.


## CHAPTER 3 Describing Data: Numerical Measures

- Updated examples.
- Revised example/solution showing the calculation of the sample mean.
- Revised example/solution showing the calculation of the geometric mean.
- Revised example/solution showing the calculation of the sample standard deviation.
- Updated Statistics in Action with most recent Major League Baseball data.
- The section of the relative positions of the mean, median, and mode as well as the concept of skewness have been rewritten with new illustrations.
- Updated exercises 21, 23, 25, 30, 31, 32, 33, 34, 73, 77, 82.


## CHAPTER 4 Describing Data: Displaying and Exploring Data

- Updated examples.
- Revised section on computing quartiles.
- Box plots are generated with Excel with accompanying Connect tutorial.
- Updated exercise 16 with Boston Red Sox salary data.
- Updated Self-Review $4-5$ to include data.
- A new section on the correlation coefficient added to the major section, "Describing the Relationship between Two Variables."
- Exercises 17, 18, and 32 provide data and ask the user to produce scatter plots with statistical software.
- New exercise 24.


## CHAPTER 5 A Survey of Probability Concepts

- Updated examples.
- New exercises 90, 93.
- Updated exercises 9, 10, 58.
- Revised example/solution showing the calculation of empirical probability.
- New "Principles of Counting" example/solution based on blood type.


## CHAPTER 6 Discrete Probability Distributions

- Updated examples.
- New exercises 9, 10, 17, 18, 24, 25, 49, 66, 67, 68, 69.
- Updated exercise 5, 7, 8, 24, 58.
- New example/solution using the binomial distribution to predict purchase behavior with debit and credit cards.


## CHAPTER 7 Continuous Probability Distributions

- New opening page and exercise.
- Revised example/solution demonstrating the uniform probability distribution.
- For all uniform distribution examples and exercises, users are asked to compute the mean and standard deviation.
- Updated and revised exercise 1, 2, 5, 7, 8, 11, 12, 28.
- New exercises 15, 19, 20, 35, 36, 41, 43, 47, 55, 61.
- Added emphasis on defining the probability of a particular value of a continuous random variable to be zero.
- Removed the section: "The Normal Approximation to the Binomial." Using statistical software, there is no need for approximations.


## CHAPTER 8 Sampling, Sampling Methods, and the

 Central Limit Theorem- New chapter title: "Sampling, Sampling Methods, and the Central Limit Theorem."
- New section titled: "Research and Sampling." This section now introduces the idea of sampling by discussing the research process (i.e., posing a research question, collecting data, processing the data, and making data-based inferences and conclusions). The section also discusses ethical guidelines for selecting representative samples from populations.
- Revised Self-Review 8-1.
- Revised Statistics in Action about biased sampling.
- New section: "Sample Mean as a Random Variable."
- Revised example/solution demonstrating sampling, the sample mean as a random variable, and the meaning and interpretation of sampling error.
- New section: "Standard Error of the Mean."
- New Exercises 14, 15, 16.
- Updated exercises 12, 13, 18, 19, 20, 21, 25, 26, 31, 32, 36.


## CHAPTER 9 Estimation and Confidence Intervals

- Introduction now connects Chapter 8 to Chapter 9 by discussing how sample data are used to estimate population parameters.
- More emphasis placed on the margin of error in the calculation and interpretation of a confidence interval.
- Revised example/solution demonstrating the calculation and interpretation of a confidence interval estimating a population mean.
- Revised example/solution demonstrating the calculation and interpretation of a confidence interval estimating a population proportion.
- Revised example/solution demonstrating the margin of error in computing a sample size.
- Section "Finite-Population Correction Factor" emphasizes the effect of finite populations on the margin of error.
- New exercises 17, 41, 42, 43, 44, 45, 46
- Updated exercises 5, 11, 12, 19, 22, 25, 32, 33, 35, 37, 41, $42,43,44,45,46,47,51,52,54,55,58,61$


## CHAPTER 10 One-Sample Tests of Hypothesis

- The chapter now relies on statistical software to compute $p$-values.
- Revised example/solution demonstrating the hypothesis testing procedure with added emphasis on the significance level and the Type I error probability.
- Revised Self-Review 10-1
- Revised section titled: p-value in hypothesis testing
- Revised Self-Review 10-2
- Revised example/solutions demonstrating hypothesis testing with the $t$-statistic.
- New exercises 33, 34.


## CHAPTER 11 Two-Sample Tests of Hypothesis

- Revised explanation comparing independent and dependent sampling.
- Updated exercises 4, 9.
- Exercises 17-46 are now structured with multi-item responses providing users with directed questions.


## CHAPTER 12 Analysis of Variance

- Revised Self-Review 12-2.
- Revised Self-Review 12-3
- Revised Self-Review 12-4
- Updated exercises 1-6, 9, 10, 13, 14, 29, 30, 31, 32, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45
- New Statistics in Action illustrating ANOVA.


## CHAPTER 13 Correlation and Linear <br> Regression

- Chapter encourages users to compute p-values with statistical software including on-line p-value calculators.
- New exercises 17, 35, 41, 43.
- Updated exercises 36, 47, 48, 49, 51, 52, 57, 58, 59.


## CHAPTER 14 Multiple Regression Analysis

- New Statistics in Action describing spurious correlation.
- Updated exercises 1, 2, 26, 27, 28, 30.


## CHAPTER 15 Nonparametric Methods: Nomina

 Level Hypothesis Test- Revised chapter face page.
- Revised example/solution testing two population proportions.
- New Exercises 24, 46
- Updated Exercise 31.


## CHAPTER 16 Nonparametric Methods: Analysis of Ordinal Data

- Revised chapter face page.
- Deleted section "Using the Normal Approximation to the Binomial."
- Revised example/solution: Hypothesis test of the median
- Revised example/solution: Rank correlation.
- New Exercises 4, 5, 6, 21.
- Updated exercises 28, 29.


## CHAPTER 17 Index Numbers

- Entire chapter is updated with current dates and data.


## CHAPTER 18 Forecasting with Time Series Analysis

- Completely rewritten to focus on time series forecasting.


## CHAPTER 19 Statistical Process Control and Quality Management

- Updated 2018 Malcolm Baldridge National Quality Award winners.


## CHAPTER 20 An Introduction to Decision Theory

- Updated exercises.
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## What Is Statistics?



B BEST BUY sells Fitbit wearable technology products that track a person's physical activity and sleep quality. The Fitbit technology collects daily information on a person's number of steps so that a person can track calories burned. The information can be synced with a cell phone and displayed with a Fitbit app. Assume you know the daily number of Fitbit Flex 2 units sold last month at the Best Buy store in Collegeville, Pennsylvania. Describe a situation where the number of units sold is considered a sample. Illustrate a second situation where the number of units sold is considered a population. (See Exercise 11 and LO1-3.)

## LEARNING OBJECTIVES

When you have completed this chapter, you will be able to:
LO1-1 Explain why knowledge of statistics is important.
LO1-2 Define statistics and provide an example of how statistics is applied.
L01-3 Differentiate between descriptive and inferential statistics.
L01-4 Classify variables as qualitative or quantitative, and discrete or continuous.
LO1-5 Distinguish among nominal, ordinal, interval, and ratio levels of measurement.
LO1-6 List the values associated with the practice of statistics.


Gregor Schuster/Getty Images

## L01-1

Explain why knowledge of statistics is important.


Source: Courtesy of Domo, Inc. important decision.

## Introduction

Suppose you work for a large company and your supervisor asks you to decide if a new version of a smartphone should be produced and sold. You start by thinking about the product's innovations and new features. Then, you stop and realize the consequences of the decision. The product will need to make a profit so the pricing and the costs of production and distribution are all very important. The decision to introduce the product is based on many alternatives. So how will you know? Where do you start?

Without experience in the industry, beginning to develop an intelligence that will make you an expert is essential. You select three other people to work with and meet with them. The conversation focuses on what you need to know and what information and data you need. In your meeting, many questions are asked. How many competitors are already in the market? How are smartphones priced? What design features do competitors' products have? What features does the market require? What do customers want in a smartphone? What do customers like about the existing products? The answers will be based on business intelligence consisting of data and information collected through customer surveys, engineering analysis, and market research. In the end, your presentation to support your decision regarding the introduction of a new smartphone is based on the statistics that you use to summarize and organize your data, the statistics that you use to compare the new product to existing products, and the statistics to estimate future sales, costs, and revenues. The statistics will be the focus of the conversation that you will have with your supervisor about this very

As a decision maker, you will need to acquire and analyze data to support your decisions. The purpose of this text is to develop your knowledge of basic statistical techniques and methods and how to apply them to develop the business and personal intelligence that will help you make decisions.

## Why Study Statistics?

If you look through your university catalogue, you will find that statistics is required for many college programs. As you investigate a future career in accounting, economics, human resources, finance, business analytics, or other business area, you also will discover that statistics is required as part of these college programs. So why is statistics a requirement in so many disciplines?

A major driver of the requirement for statistics knowledge is the technologies available for capturing data. Examples include the technology that Google uses to track how Internet users access websites. As people use Google to search the Internet, Google records every search and then uses these data to sort and prioritize the results for future Internet searches. One recent estimate indicates that Google processes 20,000 terabytes of information per day. Big-box retailers like Target, Walmart, Kroger, and others scan every purchase and use the data to manage the distribution of products, to make decisions about marketing and sales, and to track daily and even hourly sales. Police departments collect and use data to provide city residents with maps that communicate information about crimes committed and their location. Every organization is collecting and using data to develop knowledge and intelligence that will help people make informed decisions and track the implementation of their decisions. The graphic to the left shows the amount of data generated every minute (www.domo .com). A good working knowledge of statistics is useful for summarizing and organizing data to provide information that is useful and supportive of decision making. Statistics is used to make valid comparisons and to predict the outcomes of decisions.

## L01-2

Define statistics and provide an example of how statistics is applied.

STATISTICS IN ACTION
A feature of our textbook is called Statistics in Action. Read each one carefully to get an appreciation of the wide application of statistics in management, economics, nursing, law enforcement, sports, and other disciplines.

- In 2018, Forbes published a list of the richest Americans. Jeff Bezos of Amazon is the richest. His net worth is estimated at $\$ 160.0$ billion This year he surpassed Bill Gates of Microsoft who held the title for the previous 24 years (www.forbes.com) - In 2018, the four largest privately owned American companies, ranked by revenue, were Cargill, Koch Industries, Albertsons, and Deloitte. (www.forbes.com)
- In the United States according to March 2018 data, a typical high school graduate earns $\$ 712$ per week, a typical college graduate with a bachelor's degree earns \$1,173 per week, and a typical college graduate with a master's degree earns \$1,401 per week. (www.bls.gov/emp/ chart-unemployment-earnings-education .htm)

In summary, there are at least three reasons for studying statistics: (1) data are collected everywhere and require statistical knowledge to make the information useful, (2) statistical techniques are used to make professional and personal decisions, and (3) no matter what your career, you will need a knowledge of statistics to understand the world and to be conversant in your career. An understanding of statistics and statistical methods will help you make more effective personal and professional decisions.

## What Is Meant by Statistics?

This question can be rephrased in two, subtly different ways: what are statistics and what is statistics? To answer the first question, a statistic is a number used to communicate a piece of information. Examples of statistics are:

- The inflation rate is $2 \%$.
- Your grade point average is 3.5.
- The price of a new Tesla Model S sedan is \$79,570.

Each of these statistics is a numerical fact and communicates a very limited piece of information that is not very useful by itself. However, if we recognize that each of these statistics is part of a larger discussion, then the question "what is statistics" is applicable. Statistics is the set of knowledge and skills used to organize, summarize, and analyze data. The results of statistical analysis will start interesting conversations in the search for knowledge and intelligence that will help us make decisions. For example:

- The inflation rate for the calendar year was $0.7 \%$. By applying statistics we could compare this year's inflation rate to the past observations of inflation. Is it higher, lower, or about the same? Is there a trend of increasing or decreasing inflation? Is there a relationship between interest rates and government bonds?
- Your grade point average (GPA) is 3.5. By collecting data and applying statistics, you can determine the required GPA to be admitted to the Master of Business Administration program at the University of Chicago, Harvard, or the University of Michigan. You can determine the likelihood that you would be admitted to a particular program. You may be interested in interviewing for a management position with Procter \& Gamble. What GPA does Procter \& Gamble require for college graduates with a bachelor's degree? Is there a range of acceptable GPAs?
- You are budgeting for a new car. You would like to own an electric car with a small carbon footprint. The price, according to Kelley Blue Book, for a 2018 Tesla Model S Sedan is $\$ 75,700$. By collecting additional data and applying statistics, you can analyze the alternatives. For example, another choice is a hybrid car that runs on both gas and electricity. A 2018 Toyota Prius Four can be purchased for about $\$ 27,780$. Another hybrid, the 2018 Chevrolet Volt LT, costs $\$ 34,152$. What are the differences in the cars' specifications? What additional information can be collected and summarized so that you can make a good purchase decision?

Another example of using statistics to provide information to evaluate decisions is the distribution and market share of Frito-Lay products. Data are collected on each of the Frito-Lay product lines. These data include the market share and the pounds of product sold. Statistics is used to present this information in a bar chart in Chart 1-1. It clearly shows Frito-Lay's dominance in the potato, corn, and tortilla chip markets. It also shows the absolute measure of pounds of each product line consumed in the United States.

These examples show that statistics is more than the presentation of numerical information. Statistics is about collecting and processing information to create a conversation, to stimulate additional questions, and to provide a basis for making decisions. Specifically, we define statistics as:

STATISTICS The science of collecting, organizing, presenting, analyzing, and interpreting data to assist in making more effective decisions.


CHART 1-1 Frito-Lay Volume and Share of Major Snack Chip Categories in U.S. Supermarkets

In this book, you will learn the basic techniques and applications of statistics that you can use to support your decisions, both personal and professional. To start, we will differentiate between descriptive and inferential statistics.

## LO1-3

Differentiate between descriptive and inferential statistics.

## Types of Statistics

When we use statistics to generate information for decision making from data, we use either descriptive statistics or inferential statistics. Their application depends on the questions asked and the type of data available.

## Descriptive Statistics

Masses of unorganized data-such as the census of population, the weekly earnings of thousands of computer programmers, and the individual responses of 2,000 registered voters regarding their choice for president of the United States-are of little value as is. However, descriptive statistics can be used to organize data into a meaningful form. We define descriptive statistics as:

DESCRIPTIVE STATISTICS Methods of organizing, summarizing, and presenting
data in an informative way.

The following are examples that apply descriptive statistics to summarize a large amount of data and provide information that is easy to understand.

- There are a total of 46,837 miles of interstate highways in the United States. The interstate system represents only $1 \%$ of the nation's total roads but carries more than $20 \%$ of the traffic. The longest is I-90, which stretches from Boston to Seattle, a distance of 3,099 miles. The shortest is $1-878$ in New York City, which is 0.70 mile in length. Alaska does not have any interstate highways, Texas has the most interstate miles at 3,232, and New York has the most interstate routes with 28.
- Americans spent an average of \$143.56 on Valentine's Day-related gifts in 2018. About 15 percent of Americans purchased gifts cards for Valentine's Day. In addition, they spent an average of $\$ 5.50$ on gifts for their pets. (www.cnn.com/2018/02/14/ us/valentines-by-the-numbers-trnd/index.html)

Statistical methods and techniques to generate descriptive statistics are presented in Chapters 2 and 4. These include organizing and summarizing data with frequency distributions and presenting frequency distributions with charts and graphs. In addition, statistical measures to summarize the characteristics of a distribution are discussed in Chapter 3.

## Inferential Statistics

Sometimes we must make decisions based on a limited set of data. For example, we would like to know the operating characteristics, such as fuel efficiency measured by miles per gallon, of sport utility vehicles (SUVs) currently in use. If we spent a lot of time, money, and effort, all the owners of SUVs could be surveyed. In this case, our goal would be to survey the population of SUV owners.

POPULATION The entire set of individuals or objects of interest or the measurements obtained from all individuals or objects of interest.

However, based on inferential statistics, we can survey a limited number of SUV owners and collect a sample from the population.

SAMPLE A portion, or part, of the population of interest.

Samples often are used to obtain reliable estimates of population parameters. (Sampling is discussed in Chapter 8.) In the process, we make trade-offs between the time, money, and effort to collect the data and the error of estimating a population parameter. The process of sampling SUVs is illustrated in the following graphic. In this example, we would like to know the mean or average SUV fuel efficiency. To estimate the mean of the population, six SUVs are sampled and the mean of their MPG is calculated.


So, the sample of six SUVs represents evidence from the population that we use to reach an inference or conclusion about the average MPG for all SUVs. The process of sampling from a population with the objective of estimating properties of a population is called inferential statistics.

INFERENTIAL STATISTICS The methods used to estimate a property of a population on the basis of a sample.

Inferential statistics is widely applied to learn something about a population in business, agriculture, politics, and government, as shown in the following examples:

- Television networks constantly monitor the popularity of their programs by hiring Nielsen and other organizations to sample the preferences of TV viewers. During the week of December 3, 2018, The Tonight Show Starring Jimmy Fallon was viewed by 2.26 million people in the 18-49 age. The Late Show with Stephen Colbert led the age group with 3.23 million viewers (https://tvbythenumbers.zap2it.com/tag/ jimmy-kimmel-live-ratings/). These program ratings are used to make decisions about advertising rates and whether to continue or cancel a program.
- In 2015, a sample of U.S. Internal Revenue Service tax preparation volunteers were tested with three standard tax returns. The sample indicated that tax returns were completed with a $49 \%$ accuracy rate. In other words there were errors on about half of the returns. In this example, the statistics are used to make decisions about how to improve the accuracy rate by correcting the most common errors and improving the training of volunteers.

A feature of our text is self-review problems. There are a number of them interspersed throughout each chapter. The first self-review follows. Each self-review tests your comprehension of preceding material. The answer and method of solution are given in Appendix D. You can find the answer to the following self-review in 1-1 in Appendix D. We recommend that you solve each one and then check your answer.

## SELF-REVIEW 1-1



## The answers are in Appendix D.

The Atlanta-based advertising firm Brandon and Associates asked a sample of 1,960 consumers to try a newly developed chicken dinner by Boston Market. Of the 1,960 sampled, 1,176 said they would purchase the dinner if it is marketed.
(a) Is this an example of descriptive statistics or inferential statistics? Explain.
(b) What could Brandon and Associates report to Boston Market regarding acceptance of the chicken dinner in the population?

## Types of Variables

There are two basic types of variables: (1) qualitative and (2) quantitative (see Chart $1-2$ ). When an object or individual is observed and recorded as a nonnumeric characteristic, it is a qualitative variable or an attribute. Examples of qualitative variables


CHART 1-2 Summary of the Types of Variables When a variable is qualitative, we usually count the number of observations for each category and determine what percent are in each category. For example, if we observe variable eye color, what percent of the population has blue eyes and what percent has brown eyes? If the variable is type of vehicle, what percent of the total number of cars sold last month were SUVs? Qualitative variables are often summarized in charts and bar graphs (Chapter 2).

When a variable can be reported numerically, it is called a quantitative variable. Examples of quantitative variables are the balance in your checking account, the number of gigabytes of data used on your cell phone plan last month, the life of a car battery (such as 42 months), and the number of people employed by a company.

Quantitative variables are either discrete or continuous. Discrete variables can assume only certain values, and there are "gaps" between the values. Examples of discrete variables are the number of bedrooms in a house (1, 2, 3, 4, etc.), the number of cars (326, 421, etc.) arriving at Exit 25 on I-4 in Florida near Walt Disney World in an hour, and the number of students in each section of a statistics course ( 25 in section A, 42 in section $B$, and 18 in section C). We count, for example, the number of cars arriving at Exit 25 on I-4, and we count the number of statistics students in each section. Notice that a home can have 3 or 4 bedrooms, but it cannot have 3.56 bedrooms. Thus, there is a "gap" between possible values. Typically, discrete variables are counted.

Observations of a continuous variable can assume any value within a specific range. Examples of continuous variables are the air pressure in a tire and the weight of a shipment of tomatoes. Other examples are the ounces of raisins in a box of raisin bran cereal and the duration of flights from Orlando to San Diego. Grade point average (GPA) is a continuous variable. We could report the GPA of a particular student as 3.2576952 . The usual practice is to round to 3 places-3.258. Typically, continuous variables result from measuring.

## LO1-5

Distinguish among nominal, ordinal, interval, and ratio levels of measurement.

## Levels of Measurement

Data can be classified according to levels of measurement. The level of measurement determines how data should be summarized and presented. It also will indicate the type of statistical analysis that can be performed. Here are two examples of the relationship between measurement and how we apply statistics. There are six colors of candies in a bag of M\&Ms. Suppose we assign brown a value of 1 , yellow 2, blue 3, orange 4 , green 5 , and red 6 . What kind of variable is the color of an M\&M? It is a qualitative variable. Suppose someone summarizes M\&M color by adding the assigned color values, divides the sum by the number of M\&Ms, and reports that the mean color is 3.56 . How do we interpret this statistic? You are correct in concluding that it has no meaning as a measure of M\&M color. As a qualitative variable, we can only report the count and percentage of each color in a bag of M\&Ms. As a second example, in a high school track meet there are eight competitors in the 400-meter run. We report the order of finish and that the mean finish is 4.5. What does the mean finish tell us? Nothing! In both of these instances, we have not used the appropriate statistics for the level of measurement.
There are four levels of measurement: nominal, ordinal, interval, and ratio. The lowest, or the most primitive, measurement is the nominal level. The highest is the ratio level of measurement.

## Nominal-Level Data

For the nominal level of measurement, observations of a qualitative variable are measured and recorded as labels or names. The labels or names can only be classified and counted. There is no particular order to the labels.

NOMINAL LEVEL OF MEASUREMENT Data recorded at the nominal level of measurement is represented as labels or names. They have no order. They can only be classified and counted.

A classification of M\&M candies based on their color is an example of the nominal level of measurement. We simply classify the candies by color. There is no natural order. That is, we could report the brown candies first, the orange first, or any of the other colors first. Recording the variable gender is another example of the nominal level of measurement. Suppose we count the number of students entering a football game with a student ID and report how many are men and how many are women. We could report either the men or the women first. For the data measured at the nominal level, we are limited to counting the number in each category of the variable. Often, we convert these counts to percentages. For example, a random sample of M\&M candies reports the following percentages for each color:

| Color | Percent in a bag |
| :--- | :---: |
| Blue | $24 \%$ |
| Green | $20 \%$ |
| Orange | $16 \%$ |
| Yellow | $14 \%$ |
| Red | $13 \%$ |
| Brown | $13 \%$ |

To process the data for a variable measured at the nominal level, we often numerically code the labels or names. For example, if we are interested in measuring the home state for students at East Carolina University, we would assign a student's home state of Alabama a code of 1 , Alaska a code of 2 , Arizona a 3, and so on. Using this procedure with an alphabetical listing of states, Wisconsin is coded 49 and Wyoming 50. Realize that the number assigned to each state is still a label or name. The reason we assign numerical codes is to facilitate counting the number of students from each state with statistical software. Note that assigning numbers to the states does not give us license to manipulate the codes as numerical information. Specifically, in this example, $1+2=3$ corresponds to Alabama + Alaska = Arizona. Clearly, the nominal level of measurement does not permit any mathematical operation that has any valid interpretation.

## Ordinal-Level Data

The next higher level of measurement is the ordinal level. For this level of measurement a qualitative variable or attribute is either ranked or rated on a relative scale.

ORDINAL LEVEL OF MEASUREMENT Data recorded at the ordinal level of measurement is based on a relative ranking or rating of items based on a defined attribute or qualitative variable. Variables based on this level of measurement are only ranked or counted.

For example, many businesses make decisions about where to locate their facilities; in other words, where is the best place for their business? Business Facilities (Search "Rankings" at https://businessfacilities.com/) publishes a list of the top 10 states for the "best business climate." The 2018 rankings are shown to the left. They are based on the evaluation of many different factors, including the cost of labor, business tax climate, quality of life, transportation infrastructure, educated workforce, and economic growth potential.

This is an example of an ordinal scale because the states are ranked in order of best to worst business climate. That is, we know the relative order of the states based on the attribute. For example, in 2018 Alabama had the best business climate and Texas was second. Virginia was fifth, and that was better than South Carolina but not as good as Utah. We cannot say that Alabama's business climate is five times better than Virgina's business climate because the magnitude of the difference between the states is not known. To put it another way, we do not know if the magnitude of the difference between Alabama and Texas is the same as between Tennessee and Utah.

Another example of the ordinal level measure is based on a scale that measures an attribute. This type of scale is used when students rate instructors on a variety of attributes. One attribute may be: "Overall, how do you rate the quality of instruction in this class?" A student's response is recorded on a relative scale of inferior, poor, good, excellent, and superior. An important characteristic of using a relative measurement scale is that we cannot distinguish the magnitude of the differences between the responses. We do not know if the difference between "Superior" and "Good" is the same as the difference between "Poor" and "Inferior."

Table 1-1 lists the frequencies of 60 student ratings of instructional quality for Professor James Brunner in an Introduction to Finance course. The data are summarized based on the order of the scale used to rate the instructor. That is, they are summarized by the number of students who indicated a rating of superior (6), good (26), and so on. We also can convert the frequencies to percentages. About 43.3\% (26/60) of the students rated the instructor as good.

TABLE 1-1 Rating of a Finance Professor

| Rating | Frequency | Percentage |
| :--- | :---: | :---: |
| Superior | 6 | $10.0 \%$ |
| Good | 26 | $43.3 \%$ |
| Average | 16 | $26.7 \%$ |
| Poor | 9 | $15.0 \%$ |
| Inferior | 3 | $5.0 \%$ |

## Interval-Level Data

The interval level of measurement is the next highest level. It includes all the characteristics of the ordinal level, but, in addition, the difference or interval between values is meaningful.

INTERVAL LEVEL OF MEASUREMENT For data recorded at the interval level of measurement, the interval or the distance between values is meaningful. The interval level of measurement is based on a scale with a known unit of measurement.

The Fahrenheit temperature scale is an example of the interval level of measurement. Suppose the high temperatures on three consecutive winter days in Boston are 28, 31, and 20 degrees Fahrenheit. These temperatures can be easily ranked, but we can also determine the interval or distance between temperatures. This is possible because 1 degree Fahrenheit represents a constant unit of measurement. That is, the distance between 10 and 15 degrees Fahrenheit is 5 degrees, and is the same as the 5 -degree distance between 50 and 55 degrees Fahrenheit. It is also important to note that 0 is just a point on the scale. It does not represent the absence of the condition. The measurement of zero degrees Fahrenheit does not represent the absence of heat or cold. But by our own measurement scale, it is cold! A major limitation of a variable measured at the interval level is that we cannot make statements similar to 20 degrees Fahrenheit is twice as warm as 10 degrees Fahrenheit.

